The University of Alabama at Birmingham (UAB)
Department of Physics

PH 461/561 – Classical Mechanics I – Fall 2005

Assignment # 10   Due: **Tuesday, November 29**

1. A particle of mass m moves in one dimension under a restoring force $-kx$, a linear resistance $-b\dot{x}$, and a time-dependent force $F_0 \cos \omega t$. The quantities $k,\ b,\ F_0,$ and ω are all positive constants, with $k = b^2/4m$.
   (a) Find $x(t)$ and show that it may be understood as the sum of a *transient term* that vanishes for $t \to \infty$, and a *steady state term* that dominates the motion when $t \to \infty$.
   (b) Graph the behavior of the amplitude of the *steady state term* as a function of the frequency ω, for various values of the damping factor $\gamma \equiv b/2m$.
   (c) Discuss the effect of the damping factor γ on the sharpness of the resonance.
   (d) Graph the behavior of the phase difference between the *steady state term* and the force $F_0 \cos \omega t$ as a function of the frequency ω, for various values of the damping factor γ.
   (e) Discuss the physical meaning of the phase difference dependence on ω, with particular attention to the cases when $\omega \to 0$ (low frequency regime) and $\omega \to \infty$ (high frequency regime).

2. Given a force $\mathbf{F} = F_x\,\mathbf{i} + F_y\,\mathbf{j} + F_z\,\mathbf{k}$, with components

$$F_x = a y^2 z^3 \quad \text{and} \quad F_y = 2a xyz^3$$

   (a) Determine a component F_z such that the force is conservative.
   (b) In this case, calculate the potential $V(x,y,z)$ such that $V(0,y,z) = 0$.

3. A particle of mass m moves in three dimensions under a time-dependent force whose components are

$$F_x = a, \quad F_y = bt, \quad F_z = ct^2$$

   where a, b, c are positive constants and t is the time.
   Provide the following:
   (a) A discussion of the degrees of freedom of the system.
   (b) An identification of any constraints to the motion of the particle.
   (c) The differential equations of motion in a suitable coordinate system. Is this force conservative? Why?
   (e) Assuming that at $t = 0$ the particle had the following initial conditions: $\boldsymbol{r}_0 = 0$ and $\boldsymbol{v}_0 = v_{0x}\mathbf{i}$, determine the motion of the particle (i.e., Find $\boldsymbol{r}(t)$)

4. A particle of mass *m* moves in three dimensions as it is subjected to the force $\vec{F} = -mg\vec{k}$, where *g* is the acceleration due to gravity. Neglecting air resistance provide the following:

   (a) A discussion of the degrees of freedom of the system.
   (b) An identification of any constraints to the motion of the particle.
   (c) The differential equations of motion in a suitable coordinate system.
   (d) Find the motion (i.e., solve the equations of motion) assuming initial conditions $\vec{r}_0 = 0$, $\vec{v}_0 \neq 0$.
   (e) Find an analytical expression for the trajectory of the particle.
   (f) Find, in terms of the given initial conditions, the maximum height the particle reaches.
   (g) Find the range of the particle (i.e., the maximum linear distance the particle reaches on the *x*-*y* plane).

5. Re-work problem 2 above assuming that in addition to the force of gravity $\vec{F}$, the particle is also subject to a linear air resistance $\vec{F}_{air} = -b(d\vec{r}/dt)$.

6. Consider a particle of mass *m* in three dimensions, subject to a restoring force that may be expressed in Cartesian coordinates as $\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$, where $F_x = -k_x x;\quad F_y = -k_y y;\quad F_z = -k_z z.$ (The positive constants $k_x,\quad k_y,\quad k_z$ may or may not be equal).

   (a) Find an expression for the potential energy of the particle.
   (b) Find an expression for the total mechanical energy of the particle.
   (c) Is this force conservative? Why?

7. A particle of mass *m* is constrained to move in two dimensions under the force $\vec{F} = -k\vec{r}$ where *k* is a constant and $\vec{r}$ is the position vector of the particle with respect to the origin. Provide the following:

   (a) A discussion of the degrees of freedom of the system.
   (b) An identification of any constraints to the motion of the particle.
   (c) The differential equations of motion in a suitable coordinate system.
   (d) Find the motion (i.e., solve the equations of motion) assuming initial position $\vec{r}_0 = y_0\vec{j}$ and initial velocity $\vec{v}_0 = v_0\vec{i}$, where $\vec{i}, \vec{j}$ are the unit vectors in the *x*, *y* directions.
   (e) Find an analytical expression for the trajectory of the particle.

8. A particle of mass m is constrained to move in two dimensions under the force $\vec{F} = -(k_x x)\vec{i} - (k_y y)\vec{j}$ where k_x, k_y are positive constants. The particle is launched with initial conditions $\vec{r}_0 = x_0\vec{i}$ and $\vec{v}_0 = v_0\vec{j}$. Find $x(t)$, $y(t)$, and plot the particle trajectory for the three cases:
   (a) $k_y = k_x$
   (b) $k_y = (4/9)k_x$
   (c) $k_y = (1/\pi)k_x$
   (d) Discuss the differences and similarities of the trajectories you plotted in parts (a), (b), (c).

   <u>*Notes:*</u>
   I. Make sure you find the constants in the expressions for $x(t)$ and $y(t)$ in terms of the given initial conditions.
   II. The trajectories for parts (b) and (c) cannot be expressed in analytical form. Generate numerical plots of these trajectories.

9. A particle of mass m and charge q moves under the effect of a static and uniform electric field that points in the positive z direction (i.e., $\vec{E} = E_0\vec{k}$). It the particle has initial position $\vec{r}_0 = (x_0, y_0, z_0)$ and initial velocity $\vec{v}_0 = (\dot{x}_0, \dot{y}_0, \dot{z}_0)$, find its motion.

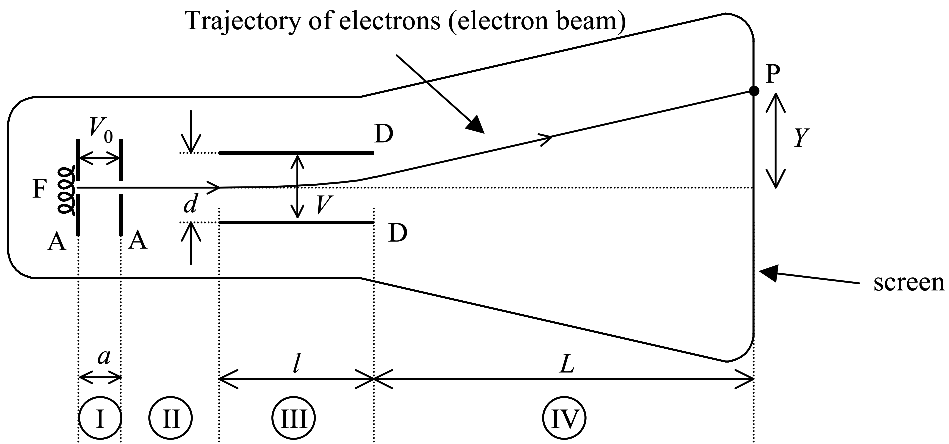
10. A particle of charge q and mass m is at rest in a magnetic field $\vec{B} = \vec{k}B_0$. At time $t = 0$, an oscillating electric field $\vec{E}(t) = \vec{j}E_0 \sin \omega t$ is turned on.

    a) Determine and discuss the motion of the particle.
    b) Determine and discuss the motion of the particle when $\omega = qB_0/m$

11. The figure in the next page shows the schematic of a cathode ray tube. Electrons (mass m, charge e) generated at the heated filament F are subject to controlled electrical forces in an evacuated chamber and hit the phosphor screen at a desired point P. Constant voltages V_0 and V are applied to plates AA and DD, respectively. Assume that:
    (a) Upon emission, electrons are at rest in the vicinity of plate A next to filament F.
    (b) Electrical forces in region II are negligible
    (c) The magnitude of the force on a particle with charge q moving between parallel plates separated by a distance r and subject to a potential difference U is given by:

$$Force = \frac{qU}{r}$$

    (d) The direction of this force is perpendicular to the plane of the plates.



a. Find the horizontal and vertical components of the electron acceleration in regions I, II, III, and IV.

b. Find the horizontal and vertical components of the electron velocity in regions I, II, III, and IV.

c. Calculate the vertical deflection Y on the tube screen with respect to the initial direction of propagation of the electrons.